Motivation

- we assume a body that is rotated about the $\boldsymbol{e}_3$-axis by the angle φ
- the deformation gradient $\boldsymbol{F}$ is then given as follows
  (in terms of the component matrix w.r.t. an orthonormal basis)

$$F_{ij} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- so $\boldsymbol{F}$ is represented by a versor
- subsequent displacement gradient $\boldsymbol{H} = \boldsymbol{F} - \boldsymbol{1}$ results ($\boldsymbol{1} = \boldsymbol{e}_i \otimes \boldsymbol{e}_i$)

$$H_{ij} = \begin{bmatrix} \cos\varphi - 1 & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- in a geometrically linear setting, the linearized and symmetrized strain tensor $\boldsymbol{E} = \mathrm{sym}(\boldsymbol{H})$ is applied

$$\boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{H} + \boldsymbol{H}^\top\right)$$

- in present case, the following results

$$E_{ij} = \begin{bmatrix} \cos\varphi - 1 & 0 & 0 \\ 0 & \cos\varphi - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- the trace of the strain tensor represents the volume change

$$\mathrm{tr}(\boldsymbol{E}) = 2(\cos\varphi - 1)$$

- when consulting Hooke's law, stresses $\boldsymbol{T}$ would arise

$$\boldsymbol{T} = \lambda\,\mathrm{tr}(\boldsymbol{E})\boldsymbol{1} + 2\mu\boldsymbol{E}$$

  (herein, $\lambda = K - {}^2\!/\!_3 G$ and $\mu = G$ are material parameters)
- this is not true, of course, since the strain tensor $\boldsymbol{E}$ is only valid for small angles φ, so that the following holds

$$|\varphi| \ll 1 \qquad \Rightarrow \qquad \cos\varphi \approx 1$$

- if, and only if, this small-angle approximation holds, $\boldsymbol{E} \approx \boldsymbol{0}$ and $\mathrm{tr}(\boldsymbol{E}) = 0$ result in present case, consequently no stresses arise
- the geometrically linear theory only applies under the assumption of small deformations and small rotations
- for all other cases, a geometrically non-linear approach is therefore necessary