St. Venant-Kirchhoff-**Materialgesetz**

$$\overset{\text{2PK}}{\boldsymbol{T}} = \lambda I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} + 2\mu\boldsymbol{E}^{\text{G}}$$

Ges.: Elastische Energie w, die zu diesem Konstitutivgesetz gehört

Lösung:

$$\text{Ansatz} \Rightarrow \overset{\text{2PK}}{\boldsymbol{T}} = \frac{\partial w}{\boldsymbol{E}^{\text{G}}} = \frac{\partial w}{\partial I_{\boldsymbol{E}^{\text{G}}}}\frac{\partial I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}} + \frac{\partial w}{\partial I\!I_{\boldsymbol{E}^{\text{G}}}}\frac{\partial I\!I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}}$$

$$\frac{\partial w}{\partial I_{\boldsymbol{E}^{\text{G}}}} = 2\mu I_{\boldsymbol{E}^{\text{G}}} + \lambda I_{\boldsymbol{E}^{\text{G}}} = (2\mu+\lambda) I_{\boldsymbol{E}^{\text{G}}} \qquad \frac{\partial I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}} = \mathbf{1}$$

$$\Rightarrow w_I = \left(\mu + \frac{1}{2}\lambda\right) I^2_{\boldsymbol{E}^{\text{G}}}$$

$$\frac{\partial w}{\partial I\!I_{\boldsymbol{E}^{\text{G}}}} = -2\mu \qquad \frac{\partial I\!I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}} = I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} - \boldsymbol{E}^{\text{G}}$$

$$\Rightarrow w_{I\!I} = -2\mu I\!I_{\boldsymbol{E}^{\text{G}}}$$

$$w = w_{I\&I\!I} = -2\mu I\!I_{\boldsymbol{E}^{\text{G}}} + \left(\mu + \frac{1}{2}\lambda\right) I^2$$

Kontrolle:

$$\begin{aligned}\overset{\text{2PK}}{\boldsymbol{T}} &= \frac{\partial w}{\partial I_{\boldsymbol{E}^{\text{G}}}}\frac{\partial I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}} + \frac{\partial w}{\partial I\!I_{\boldsymbol{E}^{\text{G}}}}\frac{\partial I\!I_{\boldsymbol{E}^{\text{G}}}}{\partial \boldsymbol{E}^{\text{G}}}\\ &= (2\mu+\lambda) I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} - 2\mu(I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} - \boldsymbol{E}^{\text{G}})\\ &= \cancel{2\mu I_{\boldsymbol{E}^{\text{G}}}\mathbf{1}} + \lambda I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} \cancel{- 2\mu I_{\boldsymbol{E}^{\text{G}}}\mathbf{1}} + 2\mu\boldsymbol{E}^{\text{G}}\\ &= \lambda I_{\boldsymbol{E}^{\text{G}}}\mathbf{1} + 2\mu\boldsymbol{E}^{\text{G}}\end{aligned}$$