
CIARLET-**Energie**

Spannungen infolge vorgegebener Deformationen plotten.

$$\boldsymbol{T} = \frac{1}{J}\left[\mu\,\boldsymbol{F}\boldsymbol{F}^T + \left(\frac{\lambda}{2}(J^2-1) - \mu\right)\mathbf{1}\right]$$

LAMÉsche Parameter: λ, μ Eigenwerte von $\boldsymbol{F}$: α JACOBI-Determinante: J

1. Fall

$\boldsymbol{F} = \alpha\,\mathbf{1}$ Ges.: $T_{11}(\alpha)$

$$\begin{aligned}
J &= \alpha^3 \\
\boldsymbol{T} &= \frac{1}{\alpha^3}\left[\mu\,\alpha^2\mathbf{1} + \left(\frac{\lambda}{2}(\alpha^6-1) - \mu\right)\mathbf{1}\right] \\
\boldsymbol{T} &= \frac{\mathbf{1}}{\alpha^3}\left[\mu\,\alpha^2 + \frac{\lambda}{2}\alpha^6 - \frac{\lambda}{2} - \mu\right] \\
\boldsymbol{T} &= \frac{\mathbf{1}}{\alpha^3}\left[\mu(\alpha^2-1) + \frac{\lambda}{2}(\alpha^6-1)\right] \\
T_{11} &= \frac{\mu(\alpha^2-1) + \frac{\lambda}{2}(\alpha^6-1)}{\alpha^3}
\end{aligned}$$

2. Fall

$$\boldsymbol{F} = \begin{bmatrix} \alpha & & \\ & 1 & \\ & & 1 \end{bmatrix} \boldsymbol{e}_i \otimes \boldsymbol{e}_j$$ Ges.: $T_{11}(\alpha)$ & $T_{22}(\alpha)$

$$\begin{aligned}
J &= \alpha \\
\boldsymbol{T} &= \frac{1}{\alpha}\left[\mu\left[\begin{matrix} \alpha & & \\ & 1 & \\ & & 1 \end{matrix}\right]^2 + \left(\frac{\lambda}{2}(\alpha^2-1) - \mu\right)\mathbf{1}\right] \\
\boldsymbol{T} &= \frac{1}{\alpha}\left[\mu\left(\left[\begin{matrix} \alpha & & \\ & 1 & \\ & & 1 \end{matrix}\right]^2 - \mathbf{1}\right) + \frac{\lambda\mathbf{1}}{2}(\alpha^2-1)\right] \\
T_{11} &= \frac{\mu(\alpha^2-1) + \frac{\lambda}{2}(\alpha^2-1)}{\alpha} \\
T_{22} &= \frac{\frac{\lambda}{2}(\alpha^2-1)}{\alpha}
\end{aligned}$$

3. Fall

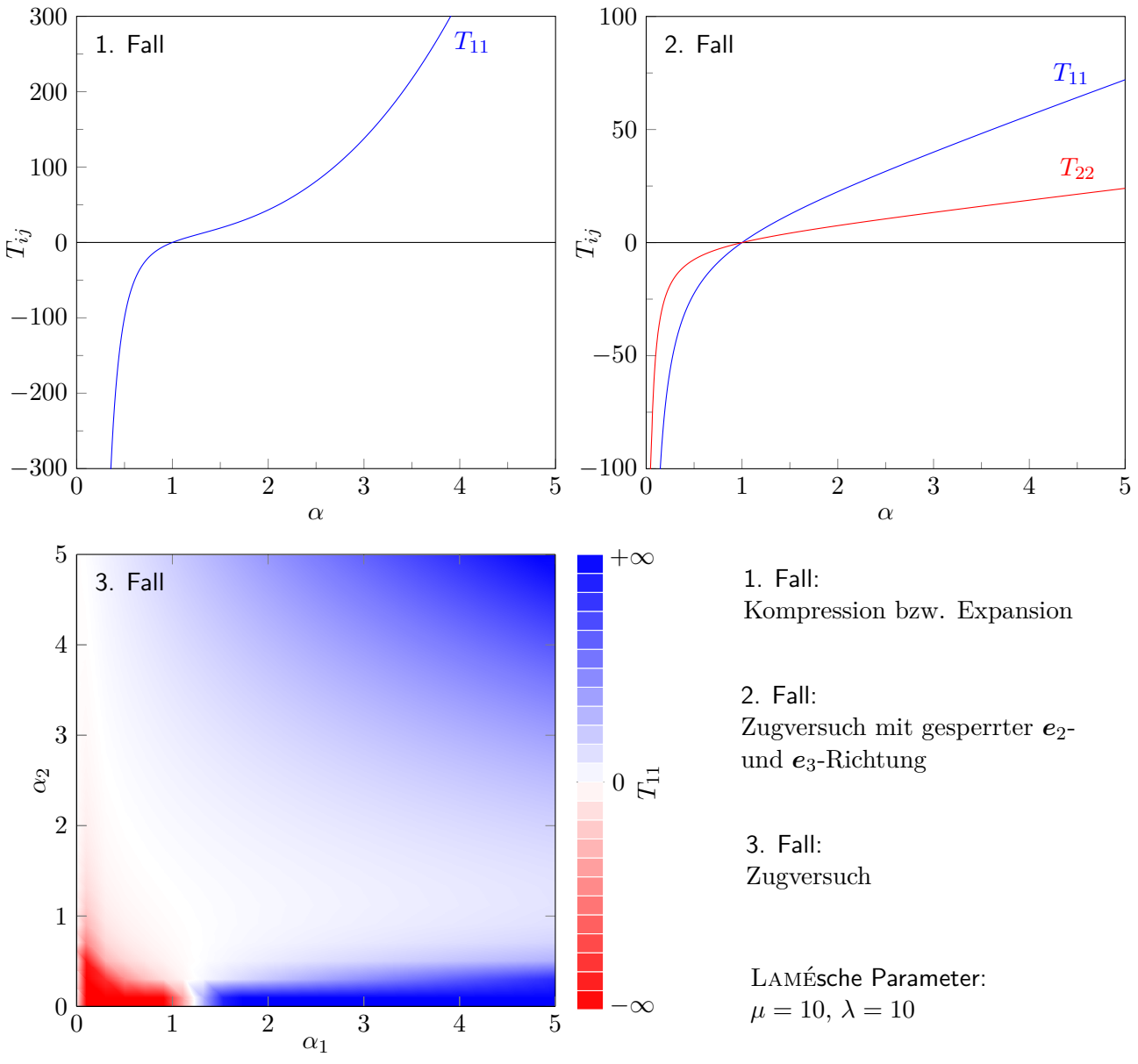
$$\boldsymbol{F} = \begin{bmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \alpha_2 \end{bmatrix} \boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad \text{Ges.: } T_{11}(\alpha_1, \alpha_2)$$

$$J = \alpha_1 \alpha_2^2$$

$$\boldsymbol{T} = \frac{1}{\alpha_1 \alpha_2^2} \left[\mu \begin{bmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \alpha_2 \end{bmatrix}^2 + \left(\frac{\lambda}{2}(\alpha_1^2 \alpha_2^4 - 1) - \mu \right) \mathbf{1} \right]$$

$$\boldsymbol{T} = \frac{1}{\alpha_1 \alpha_2^2} \left[\mu \left(\begin{bmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \alpha_2 \end{bmatrix}^2 - \mathbf{1} \right) + \frac{\lambda}{2}(\alpha_1^2 \alpha_2^4 - 1) \right]$$

$$T_{11} = \frac{\mu(\alpha_1^2 - 1) + \frac{\lambda}{2}(\alpha_1^2 \alpha_2^4 - 1)}{\alpha_1 \alpha_2^2}$$



1. Fall:
Kompression bzw. Expansion

2. Fall:
Zugversuch mit gesperrter $\boldsymbol{e}_2$- und $\boldsymbol{e}_3$-Richtung

3. Fall:
Zugversuch

Lamésche Parameter:
$\mu = 10$, $\lambda = 10$