# Exercise 06

## 1. Fourth-order basis tensors

### Problem

- A transversely isotropic material is given by the aid of the tetrad $\mathbb{C}$ possessing major and left and right minor symmetries.

$$\mathbb{C} = \sum_{I=1}^{5} C_I \mathbb{B}_I \ .$$

- Herein $C_I$ are material parameters and $\mathbb{B}_I$ are orthonormal basis tensors.

$$\begin{aligned}
\mathbb{B}_1 &= \hat{\mathbb{B}}_2 - 2\hat{\mathbb{B}}_3 - \frac{1}{2}(\hat{\mathbb{B}}_1 - \hat{\mathbb{B}}_4 - \hat{\mathbb{B}}_5) \\
\mathbb{B}_2 &= 2(\hat{\mathbb{B}}_3 - \hat{\mathbb{B}}_5) \\
\mathbb{B}_3 &= \frac{\sqrt{2}}{2}(2\hat{\mathbb{B}}_5 - \hat{\mathbb{B}}_4) \\
\mathbb{B}_4 &= \frac{1}{2}(\hat{\mathbb{B}}_1 - \hat{\mathbb{B}}_4 - \hat{\mathbb{B}}_5) \\
\mathbb{B}_5 &= \frac{1}{2}(\hat{\mathbb{B}}_1 - \hat{\mathbb{B}}_4 + 3\hat{\mathbb{B}}_5)
\end{aligned}$$

- Therein $\hat{\mathbb{B}}_I$ are non-orthonormal basis tensors, given as follows wherein $\boldsymbol{n}$ is the normal vector.

$$\begin{aligned}
\hat{\mathbb{B}}_1 &= \boldsymbol{I} \otimes \boldsymbol{I} \\
\hat{\mathbb{B}}_2 &= \mathbb{I}^{\text{sym}} \\
\hat{\mathbb{B}}_3 &= (\boldsymbol{n} \otimes \boldsymbol{I} \otimes \boldsymbol{n})^{\text{sym}} \\
\hat{\mathbb{B}}_4 &= \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{n} \otimes \boldsymbol{n} \\
\hat{\mathbb{B}}_5 &= \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n}
\end{aligned}$$

- The basis tensors $\mathbb{B}_I(\boldsymbol{n})$ are isotropic tensor functions in $\boldsymbol{n}$.
- The basis $\mathbb{B}_I$ is orthogonal and normalized to length $\sqrt{2}$.

$$\mathbb{B}_I :: \mathbb{B}_J = 2\delta_{ij}$$

- The components $C_I$ are obtained as follows.

$$C_I = \frac{1}{2}\mathbb{C} :: \mathbb{B}_I$$

- Determine the projector decomposition for this transversely isotropic $\mathbb{C}$.

### Solution

- The projector representation for a transversely isotropic stiffness is defined as.

$$\mathbb{C} = \sum_{\alpha=1}^{4} \lambda_\alpha \mathbb{P}_\alpha(\zeta)$$

- Obviously, there are four eigenvalues $\lambda_\alpha$ and the parameter $\zeta$ as the fifth degree of freedom.

- The projectors arise as follows.

$$
\begin{aligned}
\mathbb{P}_1 &= \mathbb{B}_1 \\
\mathbb{P}_2 &= \mathbb{B}_2 \\
\mathbb{P}_3 &= \frac{1}{2}(\mathbb{B}_5 + (\cos\zeta\mathbb{B}_3 + \sin\zeta\mathbb{B}_4)) \\
\mathbb{P}_4 &= \frac{1}{2}(\mathbb{B}_5 - (\cos\zeta\mathbb{B}_3 + \sin\zeta\mathbb{B}_4))
\end{aligned}
$$

- Herein, subsequent goniometric functions hold.

$$
\cos\zeta = \frac{C_3}{\sqrt{C_3^2 + C_4^2}} \qquad \sin\zeta = \frac{C_4}{\sqrt{C_3^2 + C_4^2}}
$$

- The four distinct eigenvalues are given by:

$$
\begin{aligned}
\lambda_1 &= C_1 \\
\lambda_2 &= C_2 \\
\lambda_3 &= C_5 + \sqrt{C_3^2 + C_4^2} \\
\lambda_4 &= C_5 - \sqrt{C_3^2 + C_4^2}
\end{aligned}
$$

- One may note that the following holds.

$$
\begin{aligned}
\mathbb{I}^{\mathrm{sym}} &= \sum_{\alpha=1}^{4} \mathbb{P}_\alpha(\zeta) \\
&= \mathbb{B}_1 + \mathbb{B}_2 + \mathbb{B}_5
\end{aligned}
$$


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